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THE APPLICATION OF OPERATIONS ANALYSIS TO WEAPON SYSTEMS DEVELOPMENT

By
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UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

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ABSTRACT: A review of the methodology of operations analysis (OA) has been completed. The basic steps required to formulate and solve an OA problem have been listed and discussed in detail. These procedures have been applied to a typical tactical situation — the submarine barrier patrol. An effectiveness model for a submarine using a hypothetical mix of weapons has been generated. Kill probabilities and cost-effectiveness comparisons have been made for a variety of weapon mix possibilities.

U.S. NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

The Application of Operations Analysis to Weapon Systems Development

An Independent Exploratoy Development (IED) program has been established to investigate the application of operational analysis techniques to weapon system development. The object of the effort is to examine a variety of basic tactical situations of interest to the Navy, and prepare mathematical models to describe them. An accumulation of these models will then serve as a library of tactical information. When a new weapon system is proposed, its effectiveness can be computed for each applicable situation and then compared with competing systems or proposals. This report is the first step in achieving the stated objective.

This paper was presented to the Graduate School of the Catholic University of America in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering in March 1969. The work was completed as part of the Laboratory's Graduate Training Program. Professor Albert Preisman of the Department of Electrical Engineering at the Catholic University of America guided the work as Thesis Advisor.

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By direction

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Part I The Methodology of Operations Analysis

1. The object of this report is to discuss the techniques of Naval Operations Analysis and apply them to a hypothetical tactical situation. (Particular emphasis will be placed on the application of these techniques to weapon system development.) Ordnance laboratories are faced with the problem of selecting the best possible weapon system or set of systems from a variety of proposals. The unprecedented cost of modern ordnance precludes the development of more than a few major weapon systems at any given time. In addition, the tactical conditions under which the chosen weapons must perform can only be projected because of the long lead time from concept formulation to the deployment of operational weapons. Because of these conditions, a severe burden is imposed upon the decision-maker who must select the weapon systems most likely to assure the continuing security of the nation. Operations Analysis (OA) is an attempt to aid the decision-maker by using scientific methods to quantitatively compare competing weapon system proposals.

2. The role of the operations analyst is not to make the decision, but rather to supply the decision-maker with a tangible basis for comparison. The analyst must deal with things that can be measured, or estimated from statistical data, or computed from theoretical considerations. The decision-maker must consider not only the aspects of the problem which are quantifiable, but also those which are not. Some examples of the latter are experience, political limitations, diplomatic agreements, and human factors such as morale, training, and possible loss of life.

3. An analyst must not only draw upon past experience; he must also anticipate tactical situations that are without precedent. The weapons being developed today cannot be produced and deployed for many years. It is true that without a thorough understanding of present day fleet tactical experiences an analyst could not make a realistic projection. But a mere extrapolation of todays technology is not sufficient to assure an adequate weapon system effectiveness model. The possibility of technological break-throughs must be anticipated. Experience is a valuable teacher, but it can be expensive. It could be said that in 1940 the French Army learned valuable lessons concerning the effectiveness of highly mobile armored divisions. The tuition for their education, however, was rather more than they could afford. An analyst's model must consider not only past experience, but possible tactical innovations as well.

4. The basic steps in formulating an (OA) problem are as follows:

(a) Determine the true objectives or mission of the operation under consideration.

(b) Try to determine all possible methods for friendly forces to accomplish the mission.

(c) Try to determine all possible enemy countermeasures to thwart the mission.

(d) Estimate the relative costs and risks to friendly forces entailed by each alternative.

(e) Decide on a measure of effectiveness (MOE) to use as

a basis for quantitatively comparing competing systems.

(f) Construct a mathematical model that can describe the physical situation.

5. The basic techniques that can be used either singly or in combination for solving an (OA) problem are as follows:

(a) Theoretical analyses, such as a probabilistic description of the tactical situation.

(b) Statistical analysis of data obtained from past tactical experience. This is a valuable supplement to theoretical analyses when it is possible.

(c) Field trials at sea under circumstances which simulate the expected tactical situation: This method is usually very effective, but extremely costly. In the case of weapon proposals that are still in the feasibility stage, it is not possible.

(d) Computer simulation of the tactical situation using Monte Carlo or game theory techniques: This method may hold great promise for the future but considerable care must be exercised to ensure that the model accurately simulates physical reality.

6. Determining the true objective or mission of a particular operation is a crucial first step in any analysis. A hasty reader might conclude that this phase of the (OA) problem is rather trivial. But the problem is far more subtle and complex than it first appears. For example, consider the naval conflict between the United Kingdom and Germany during the early years of World War II. The British had to import many commodities by sea in order to continue their war effort. The Germans deployed

submarines in an attempt to halt this maritime traffic. What then was the British objective in this conflict? It could have been defined as a need to minimize merchant shipping losses. This could have been done by keeping all their ships in port and allowing themselves to be starved into submission. The solution is absurd because the stated objective is absurd. The real objective was concerned with maintaining the flow of vital supplies and assuring their continued flow into the indefinite future. This more complex objective, which is difficult to define in precise mathematical terms, is certainly not consistent with minimizing shipping losses, although of course there was some maximum tolerable rate of shipping losses associated with the problem.

7. In the example cited above, the logical fallacy in the stated objective was blatantly obvious. Unfortunately in many tactical situations the true objectives may be difficult to determine and define. The errors in their definition may be subtle and deceptive even to experienced analysts and decision-makers. Accurate determination of the true objectives can only be achieved by carefully considering the missions of the tactical forces involved. If the essential objectives are not recognized, the (OA) results will be deceptive or useless no matter how skillfully the balance of the analysis is conducted.

8. Once the true objectives of an operation have been determined, a weapon system or systems proposal may be suggested to accomplish the mission. The analyst should then list all competing proposals which could be used to accomplish the same

purpose. Also, any existing fleet capability for performing the mission should be included. The analyst must then generate a comparative analysis of each possible alternative. Considerable care must be exercised when comparing a proposed system with an already existing capability. Capability estimates for a proposed weapon system are usually based on theoretical projections, whereas an existing capability may be measured or estimated from statistical data acquired in operational use. The latter information is usually more dependable, and a very clear cut theoretical advantage is required to justify the high cost of developing a new capability.

9. When the list of competing alternatives has been compiled, the possibility of enemy countermeasures should be considered. A list of conceivable countermeasures for each weapon system proposal should be prepared. The vulnerability of a system to existing or possible enemy capabilities may become a determining factor in estimating its worth.

10. An estimate of the relative costs and risks to friendly forces entailed by each alternative should then be made. These costs should include development, evaluation, procurement, training of personnel, exercise, and maintenance. Also, the risks involved when friendly forces use the system must also be considered. These would involve the safety provisions for handling, loading, and carrying the weapon on board ship or aircraft, as well as the assurance that the weapon once launched would not circle and attack the launcher. In addition, the tactical limitations imposed on the launching vehicle when

delivering the weapon must be considered. Would these restrictions make the launcher vulnerable to enemy counterattack? All these items must be considered in evaluating each competing proposal.

11. When the assorted alternatives have been defined, the system variables that might affect the tactical outcome of an encounter should be listed. These might include the friendly ship's speed; the target's speed; the range and destructive capability of the weapon to be used; the reliability and availability of all necessary equipment, such as the weapon, ships SONAR, fire control, and communication equipment; and the probable hydrothermal or meteorological conditions. This list should provide a guide to the complexity of the mathematical model that must be prepared. It will also indicate the basic data that must either be computed or measured. The outcome of a particular operation can be viewed as a function of all the pertinent variables.

12. If a meaningful quantitative comparison is to be made between competing systems a measure-of-effectiveness (MOE) must be established. There must be some criteria by means of which fundamentally different alternatives can be measured and compared. The crucial consideration is that the (MOE) accurately reflect the true objectives of the operation. Recall the scenario of paragraph 6 above. If merchant ships were grouped in convoys and protected by destroyer escorts, one (MOE) of the destroyers might be the number of submarines sunk per month. But of course we can easily see the fallacy in this (MOE). A high kill rate would have been a poor consolation to the British if it were

accomplished only while the submarines were succeeding in destroying the vital merchant fleet. Perhaps a better (MOE) would have been the percentage of merchant ship transits accomplished without a sinking. But even this (MOE) has pitfalls. A convoy can move only as fast as the slowest ship. If a ship capable of 16 knots is forced to travel at 8 knots, the amount of cargo it can deliver per unit time is cut in half. Its effectiveness is reduced even though it is never damaged. In addition, the destroyer escort represents an investment of national resources that could have been used to increase the size of the merchant fleet. It is conceivable that a strategy of using all resources to build merchant ships and letting them sail individually with higher risk could have been the most effective possible. Even though this strategy was probably not the best, the percentage of successful ship transits was not an adequate (MOE) for this complex scenario. If valid comparisons are to be made the (MOE) must be selected with extreme care to ensure that it accurately reflects the true objective of the operation.

13. A (MOE) frequently used in weapons system development is the cost-effectiveness. It is necessary here to define the term cost-effectiveness precisely. In recent years some people have given the expression an emotional implication which is not scientifically appropriate. It has come to be associated with economizing; that is, choosing the cheapest alternative. This is not the definition used here. Rather, cost-effectiveness is an attempt to assure the most efficient possible use of resources, even if this means choosing the most expensive

alternative. Therefore, cost-effectiveness is defined as an attempt to maximize the attainment of an objective for a given cost. Or to say the same thing in another way; it is an attempt to accomplish the objective for the smallest possible cost.

14. In weapon system development the circumstances of eventual tactical use cannot be fully anticipated. Therefore a (MOE) which tries to assure minimum cost-per-kill of an expected target seems the most suitable one to use, and this is the cost-effectiveness. An analysis in these terms is necessary because in perspective military development decisions are primarily economic ones. The major military services are provided with a limited fraction of the nations resources and must accomplish their missions to the best of their ability by maximizing the efficient use of their share. Each service must decide between missiles or airplanes, aircraft carriers or submarines, tanks or artillery, and so on ad infinitum.

15. When the above steps have been satisfactorily completed a mathematical model that accurately describes the physical situation must be prepared for each competing alternative. A solution to the (OA) problem consists then of computing the (MOE) for each alternative. The results may be a complex function of many variables and tactical situations. If so, care must be taken to present the information in a graphical or tabular form that will communicate the essential information to the decision-maker clearly, accurately, and succinctly.

16. It is worth emphasizing that once the true objective has been determined, the alternatives listed, the MOE selected

and the appropriate mathematical model generated, the actual computations may seem rather trivial. Sometimes only a few carefully selected calculations will suffice as a meaningful solution. In more complicated situations, the model can be programmed for a computer and solutions can be obtained for a wide range of the significant variables. The basic point to keep in mind however, is that an elaborately programmed mathematical model can be very impressive and provide a wealth of solutions, but these solutions are worthless if the objective, alternatives, and (MOE) have not been realistically determined.

17. There is an old story about a factory that bought a fine new but very complex machine. The machine was installed and functioned beautifully. It increased the volume of production many fold for lower cost. But one day the machine broke down. The factory maintenance men worked for days, but they were unable to repair it. Finally the factory manager was forced to call in an outside consultant. The next morning the consultant arrived and spoke briefly with the maintenance men. Then he asked for a hammer. With hammer in hand he crawled under the machine and gave it one hard rap. He crawled back out, turned on the machine, and it worked perfectly. He then drove off without another word. All was well until the end of the month when the bill arrived. It was for one thousand dollars. The factory manager was stunned, so he sent back a letter asking that the bill be itemized. A few days later the itemized bill arrived. It read:

> For striking the blow with the hammer--one dollar.
> For knowing where to strike the blow--nine hundred and ninty nine dollars

In operations analysis, solving the mathematical model is striking the blow with the hammer; finding the true objective of the mission, determining all the alternatives, choosing an appropriate (MOE), and developing the correct mathematical model for each alternative is knowing where to strike the blow.

Part II -- An Example of Operational Analysis Applied to Weapon Systems Development for a Submarine Barrier Patrol

18. It is desired to apply the techniques suggested in Part I to a hypothetical operational situation of practical interest. Because of the limited scope of this study many simplifying assumptions will be made. But hopefully, some aspects of an (OA) study will be illustrated.

19. The assumed mission will be to establish a barrier that will prevent enemy submarines from transiting some narrow water passageway, such as the sea between Iceland and the United Kingdom. Such a barrier could be established by many alternative systems, for example: patrol aircraft using sonobuoys for detection, surface ships using active SONAR systems, or minefields using passive acoustic detection. This study however, will consider only a barrier maintained by attack submarines, and specifically a portion of a barrier maintained by a single submarine.

20. The patrolling submarine is assumed to be of the attack type, ideally equipped to detect and fight a duel with an enemy submarine. Its mission is to find the enemy and force him to fight. The target submarine is assumed to be equipped primarily with strategic weapons. Its mission is to assume a tactical position from which it can launch its weapons against the Continental United States (CONUS). It is less well equipped to fight a duel with an attack submarine, but it does have some defensive capability. The target submarine will, therefore, attempt to cross the barrier without being detected. If it is detected

it will try to escape rather than fight.

21. A mathematical model will be prepared to quantitatively describe this physical operation. Included will be:

(a) The following probabilities

(1) Pd the probability of detecting a target as it tries to cross the barrier.

(2) Pa the probability of achieving a tactical position that will permit an attack. There will be a different Pa for each type of weapon considered.

(3) Pc the probability that the weapon will be capable of destroying the target given that it functions reliably.

(4) Pr the reliability (and availability) of the weapon (i.e., the probability that it will operate as the designer intended throughout the attack.

(b) The average cost of making a successful attack.

(c) The risk to the friendly submarine incurred during the attack.

22. The (MOE) used in this study will be the cost-effectiveness (i.e., average cost per kill). Although a monetary value will be placed on the attacking unit, no attempt will be made to quantize the value of human life. The risk to the friendly submarine will be separately computed and tabulated. In a real strategic situation, the decision-maker would have to weigh the estimated cost-effectiveness, the possible loss of human life, and other intangible factors against the significance of the strategic threat posed by the intended target.

23. The basic technique used for the solution of this problem will be a hypothetical theoretical analysis. Statistical analysis of past data would require effort beyond the scope of this study and might necessitate its being classified. Computer simulation

of this tactical situation would be possible, but probably would not increase its usefulness. The model used therefore, will be probabilistic, with kill probability (P_k) computed as follows:

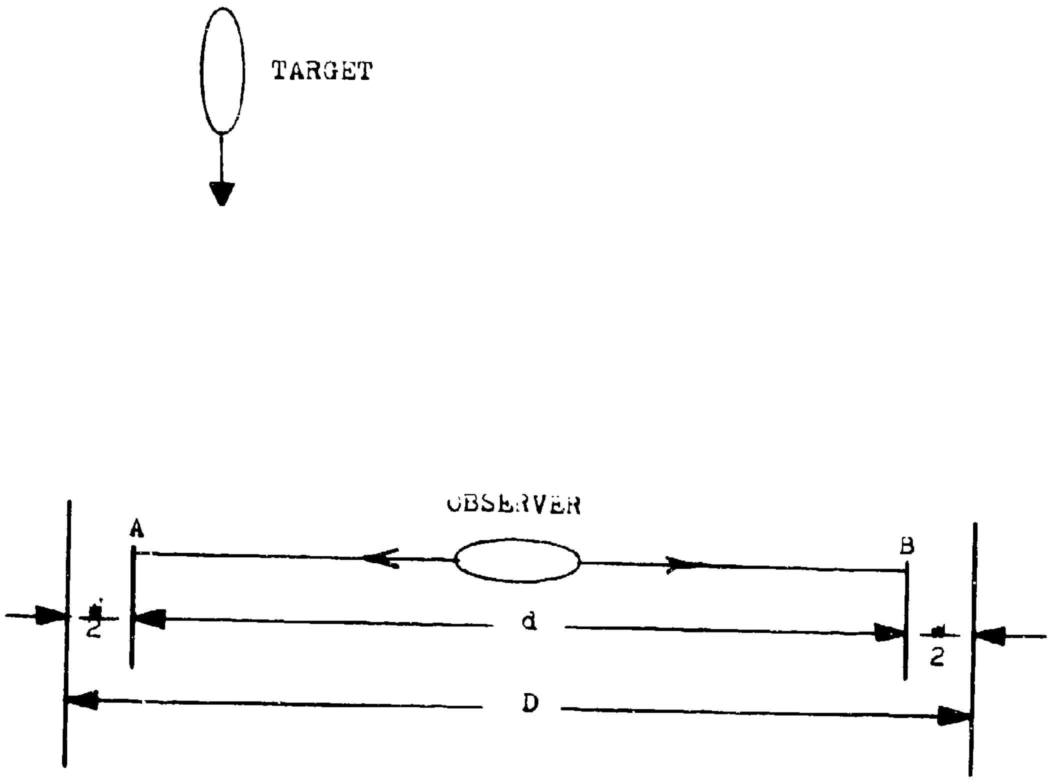
$$P_K = P_d \cdot P_a \cdot P_c \cdot P_r$$

The average cost per attack (C_a) will be computed by estimating the costs of the various weapons expended and of maintaining a submarine on patrol. The average cost per kill (C_k) can then be computed as:

$$C_K = \frac{C_a}{P_K} + C_r$$

Where C_r is the cost of risking the friendly submarine in combat.

24. Reference (a) describes a mathematical model for computing the Pd of linear patrol when the observer submarine's speed is close to the speed of the target. A brief description of this model follows. The geographic situation is illustrated in figure 1. The width of the barrier to be maintained is D. The observer submarine is assumed to have a passive detection range of $\frac{W}{2}$ a constant which is independent of both target and observer submarine speed. The observers sweep width is, therefore, W. The observer maintains the patrol by moving continuously from point A to B and back again. The target is assumed to try to cross the barrier on a path perpendicular to d at any point on path D. Any other path would increase his detection probability since he would spend more time in the danger zone. The targets



D	barrier width	$\frac{w}{2}$	passive detection range
d	observer track	u	target speed
w	sweep width	v	observer speed

Fig. 1 LINEAR BARRIER: GEOGRAPHIC CONFIGURATION

unknown location is assumed to be uniformly random with respect to both time and the crossing point of D. The targets speed is u and the observers speed is v.

25. An understanding of the relative motions of the target and the observer can be obtained by considering the following example: Visualize a Brush recorder with a paper tape and a broad line pen. The observer submarine represents the broad line pen with the sweep width proportional to the width of the pen line. The uniform probability distribution of the target is represented by the paper tape of the recorder. The paper tape moves past the pen with a speed relatively proportional to the target speed. The pen oscillates with a frequency proportional to the speed of the observer submarine. If the tape moves slowly enough the broad line pen will completely darken the paper tape. This would correspond to the case where the detection probability of Pd was one. But if the tape moves more quickly the pen will darken only a portion of it. Since the target is uniformly distributed the detection probability would simply be the darkened area divided by the total area of the tape. An example of relative observer and target movements when P (d) is less than one is shown in figure 2. The relative trace is clearly cyclical and symmetrical so that the ratio of areas of one half cycle are identical with the ratio of areas of the total trace. It is possible to express this ratio and therefore Pd in terms of the basic tactical parameters. First let us define two new symbols:

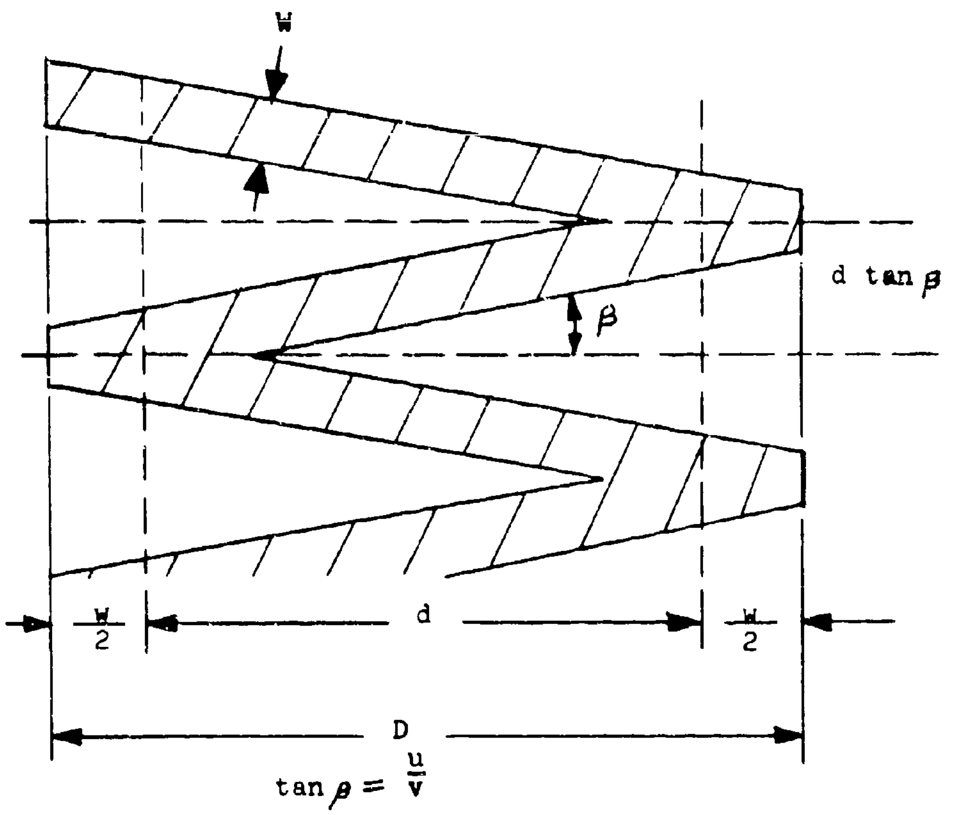


Fig. 2 Linear Barrier: **Relative Tracks**

$$\lambda = \frac{d}{w}, \qquad r = \frac{v}{u}$$

then by both references (a) and (b)

$$P_d = \left[1 - \left[\left(\lambda - \frac{\sqrt{r^2+1} - 1}{2} \right)^2 \Big/ \lambda(\lambda+1) \right] \right]$$

<u>if</u> $$r \leq 2\sqrt{\lambda(\lambda+1)}$$

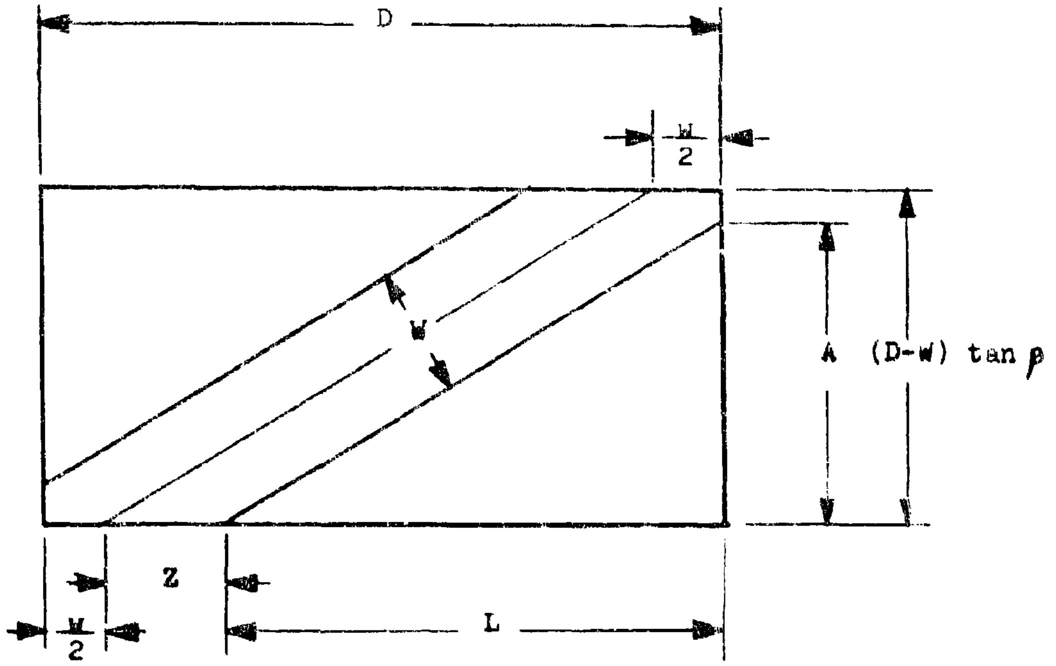
<u>if</u> $$r > 2\sqrt{\lambda(\lambda+1)}$$

<u>then</u> $$P_d = 1$$

26. Both references (a) and (b) present this equation without proof. Since the derivation may be of interest to some readers the author has generated a proof based on the geometry of the tactical situation. Because of the symmetry noted in figure 2 the proof can be based on a single half cycle. Consider the diagram of figure 3. Since the target location probability is assumed to be uniform over the entire area of the half cycle rectangle, the detection probability is simply the ratio of the swept area (shaded) to the total area. For convenience of manipulation it is desirable to let

$$P_d = 1 - P_{no}$$

where P_{no} is the probability of not detecting the target. P_{no} is equal to the area of the two unswept triangles divided by



$$Pd = \frac{\text{SHADED AREA}}{\text{TOTAL AREA}}$$

Fig. 3 DETECTION PROBABILITY

the total area. The total area (Ta) is as follows:

$$T_a = \frac{D(D-W)}{\lambda} = \frac{DW\lambda}{\lambda}$$

where D is the length of the barrier and $\frac{D-W}{r}$ is the distance the target moves while the barrier submarine is traveling the distance d-D-W. This latter is the distance the patrol submarine must travel to sweep the full length of the barrier. Note that $\frac{1}{r}$ is equal to the tangent of β. The two triangles are congruent which is evident from the symmetry of the situation. The area of one triangle $\frac{1}{2}$ LA; the area of both then is LA and

$$P_d = 1 - \frac{LA\lambda}{DW\lambda}$$

but $$L = D - \frac{W}{2} - Z$$

where $$Z = \frac{W}{2\sin\beta}$$

$$A = L\tan\beta = \left(D - \frac{W}{2} - \frac{W}{2\sin\beta}\right)\tan\beta$$

Recalling that $\tan\beta = \frac{u}{v} = \frac{1}{r}$

The area of the triangles is then

$$LA = \left(D - \frac{W}{2} - \frac{W}{2\sin\beta}\right)^2 \cdot \frac{1}{\lambda}$$

since $\tan\beta = \frac{u}{v}$, $\sin\beta = \frac{u}{\sqrt{v^2+u^2}}$

so $$P_d = 1 - \frac{\left[D - \frac{W}{2} - \frac{W}{2}\left(\frac{\sqrt{v^2+u^2}}{u}\right)\right]^2}{DW\lambda}$$

by letting $\frac{\sqrt{v^2+\mu^2}}{\mu} = \sqrt{\frac{v^2}{\mu^2}+1} = \sqrt{\lambda^2+1}$

and dividing both numerator and denominator by w^2 we obtain

$$P_d = 1 - \frac{\left[\frac{D}{W} - \frac{1}{2} - \frac{1}{2}\left(\sqrt{\lambda^2+1}\right)\right]^2}{\lambda \cdot \frac{D}{W}}$$

but $\quad \lambda = \frac{D-W}{W}$

so $\quad \frac{D}{W} = \lambda + 1$

and

$$P_d = 1 - \frac{\left(\lambda - \frac{\sqrt{\lambda^2+1}-1}{2}\right)^2}{\lambda(\lambda+1)}$$

QED

of course the equation has physical significance only when

$\lambda \leq 2\sqrt{\lambda(\lambda+1)}$ since a probability can never exceed one.

If $\lambda > 2\sqrt{\lambda(\lambda+1)}$ Then $P_d = 1$

But it is true that the time of detection is less as λ increases.

27. The above excellent model is entirely adequate for many purposes. Since this report is concerned with the relative performance of weapon systems and related SONAR and fire control systems however, the model seems inadequate. It assumes that passive detection range $\frac{W}{2}$ and therefore sweep width (W) is independent of both the target's and the observer's speed. In practice this is certainly not true. In general, the faster the target moves, the more noise it generates, and the further it can be detected. Conversely, the faster the observer moves,

the more its self noise is increased, and the more its detection range is limited. These effects are described in detail in reference (c). Thus sweep width can be more accurately represented as

$$W = f(v) + g(u)$$

where $f(v)$ is a decreasing function of v.

and $g(u)$ is an increasing function of u.

28. For generality both $f(v)$ and $g(u)$ could be represented by power series. For example:

$$f(v) = \alpha_0 + \alpha_1 v + \alpha_2 v^2 + \alpha_3 v^3 + \cdots$$

$$g(u) = \beta_0 + \beta_1 u + \beta_2 u^2 + \beta_3 u^3 + \cdots$$

where the α's and β's could be determined from the noise characteristics of the two submarines and from the detection capabilities of the observers passive SONAR. It will be assumed when v=o and u=o the submarine noise levels would be zero.

$$\therefore \quad \alpha_0 = \beta_0 = 0$$

In actual practice excellent results could probably be obtained by assuming that

$$f(v) = \alpha_1 v + \alpha_2 v^2$$

$$g(u) = \beta_1 u + \beta_2 u^2$$

However, even these simplified expressions lead to a very complex mathematical model for P (d). Since the scope of this study is severly limited and concerned primarily with weapon system models a further simplification will be made. It will be assumed that the speed ranges of both the observer and the target will be limited to 6 to 20 knots. Over this restricted speed range an interesting analysis can still be made assuming linearity such that

$$f(v) = \alpha v$$

and $$g(u) = \beta u$$

thus $$W = \alpha v + \beta u .$$

29. The new expression for W now permits a new expression for detection probability

$$d = D - W, \quad h = \frac{v}{u}, \quad \lambda = \frac{D-W}{W}$$

then

$$P_d = 1 - \frac{\left[\left(\frac{D-\alpha v-\beta u}{\alpha v+\beta u}\right) - \frac{\sqrt{\left(\frac{v}{u}\right)^2+1}-1}{2}\right]^2}{\left(\frac{D-\alpha v-\beta u}{\alpha v+\beta u}\right)\left(\frac{D-\alpha v-\beta u}{\alpha v+\beta u}+1\right)}$$

and we now have detection probability expressed as a function of target and observer speeds u and v and of barrier length D with v and u variable.

30. As described previously, α and β are constants determined by the noise levels of both the target and the observer and also by the capability of the observers passive SONAR. Hypothetical values have been assumed for these parameters. These values are not necessarily representative of actual fleet capabilities; they have been selected for purposes of illustration only. The values selected are:

$$\alpha = -0.588$$

$$\beta = +2.176$$

Table 1 reveals some of the implications of this selection for various extreme combinations of v and u.

Table 1. P (d) as a function of α and β

v	u	w	λ	r	P (d)
6	6	9.53	4.25	1.00	0.268
6	10	18.23	1.74	0.60	0.424
6	20	40.00	0.25	0.30	0.834
10	6	7.18	5.97	1.67	0.274
10	10	15.88	2.15	1.00	0.443
10	20	37.64	0.33	0.50	0.834
20	6	1.29	37.76	3.33	0.089
20	10	10.00	4.00	2.00	0.428
20	20	31.76	0.57	1.00	0.851

v = observer speed in knots

u = target speed in knots

$w = v + Bu$

$$\lambda = \frac{D-w}{w}$$

$$r = \frac{v}{u}$$

D = 50 nautical miles

$\alpha = -0.588$

$\beta = +2.176$

31. The expression P (d) has been programmed in Basic language and computations have been made for various values of the pertinent parameters. The program and a set of calculations for a barrier length (D) of 50 nautical miles have been included as enclosure (1). Sets of calculations for barrier lengths of 100 and 200 nautical miles have also been prepared and all detection probabilities are presented in matrix form in enclosure (2). It can be readily seen that for some situations an optimum observer speed lies between the limits of 6 and 20 knots. For example, with a D of 50 nautical miles and a target speed of 10 knots; the detection probability is a maximum when the observer

speed is near 14 knots.

32. The question naturally arises: for a given target speed what is the precise optimum observer speed and how should it be computed. Depending on the values of α and β it could be the minimum (6 knots) or the maximum (20 knots) for the model. However, the optimum could be between these limits as in the previous example. In this case the derivative of P (d) with respect to v would be of interest. It is as follows:

$$\frac{dP_d}{dv} = \frac{2\left(\frac{D^2}{(\alpha v+\beta u)^2}-\frac{D}{(\alpha v+\beta u)}\right)\left[\left(\frac{D}{\alpha v+\beta u}-1\right)-\frac{1}{2}\left(\left(\left(\frac{v}{u}\right)^2+1\right)^{\frac{1}{2}}-1\right)\right]\left(\frac{-\alpha D}{(\alpha v+\beta u)^2}-\frac{v}{2u^2}\left(\frac{v^2}{u^2}+1\right)^{-\frac{1}{2}}\right)}{\left(\frac{D^2}{(\alpha v+\beta u)^2}-\frac{D}{(\alpha v+\beta u)}\right)^2}$$

$$-\frac{\left[\left(\frac{D}{\alpha v+\beta u}-1\right)-\frac{1}{2}\left(\left(\left(\frac{v}{u}\right)^2+1\right)^{\frac{1}{2}}-1\right)\right]^2\cdot\left[\frac{\alpha D}{(\alpha v+\beta u)^2}-\frac{2D^2\alpha}{(\alpha u+\beta u)^3}\right]}{\left(\frac{D^2}{(\alpha v+\beta u)^2}-\frac{D}{(\alpha v+\beta u)}\right)^2}$$

setting this expression equal to zero and solving for v would be a traumatic experience, but it is not necessary. It is a comparatively simple matter to program it for the computer and solve for the zero crossings in any particular case. A copy of the program in Basic language is included as enclosure (3). In the example previously cited, the detection probability is

found to be a maximum for an observer speed between 14 and 15 knots. When the target speed increases to 12 knots, the optimum observer speed is found to be between 16 and 17 knots. These calculations suggest a possible theory of games conflict between the target and the observer.

33. However, an examination of the enclosure (2) matrices indicates that the games aspect of the present example is trivial. If the target is aware of the location of the barrier patrol its optimum strategy is always to transit at the minimum speed of 6 knots. This triviality is a direct result of letting w be a linear function of u and v. If, for example,

$$W = \alpha_1 v + \alpha_2 v^2 + \beta_1 u + \beta_2 u^2$$

a non-trivial game situation could arise between the target and the observer. A model of this type would probably reflect actual tactical situations with greater precision and would be of considerable interest to the Naval Operations Analyst. Such an effort is, however, beyond the limited scope of the present study.

34. A non trivial game theory situation may still arise from the present model if the target submarine is uncertain of the location or existence of the barrier patrol. If the target is to perform a long distance mission a speed of 6 knots, could be an untenable restriction. The target would then be forced to trade-off ocean transit time value against the risk of detection by a possible barrier patrol. In this situation the target may decide to transit at a speed well above 6 knots. Thus, the

observers detection probability would be a function of his estimate of probable target speed, and a worthwhile game situation would exist.

35. But detection of the target is only part of the problem. To use some weapons such as torpedoes the friendly submarine may have to attain a favorable position relative to the target before an attack is possible. Consider the diagram of figure 4: The friendly submarine located at point F is patrolling from west to east along the axis AB. The circle centered about F represents the area of passive detection capability with a maximum range of $\frac{W}{2}$. As before, W is the total sweep width of the barrier submarine. A target submarine is assumed to transit the barrier from north to south along the line CDE. Consider the point in time when the patrolling submarine's circle of detection is just tangent with CDE as shown in the diagram. Depending on the relative speeds of the two submarines there exists a point E such that if the target is on the south side it will cross out of the barrier zone before the advancing friendly submarine can detect it. Conversely, if it is north of E, it will be detected before it can escape from the barrier zone. In a similar manner there exists a point C such that if the target is on the north side it will not enter the barrier zone until after the friendly submarine has passed. If it is south of C it will unwittingly sail into the detection area of the patrolling submarine.

36. Let us assume that the patrolling submarine is equipped with two types of torpedoes. Also, let us assume that an ordnance development laboratory is considering the value of

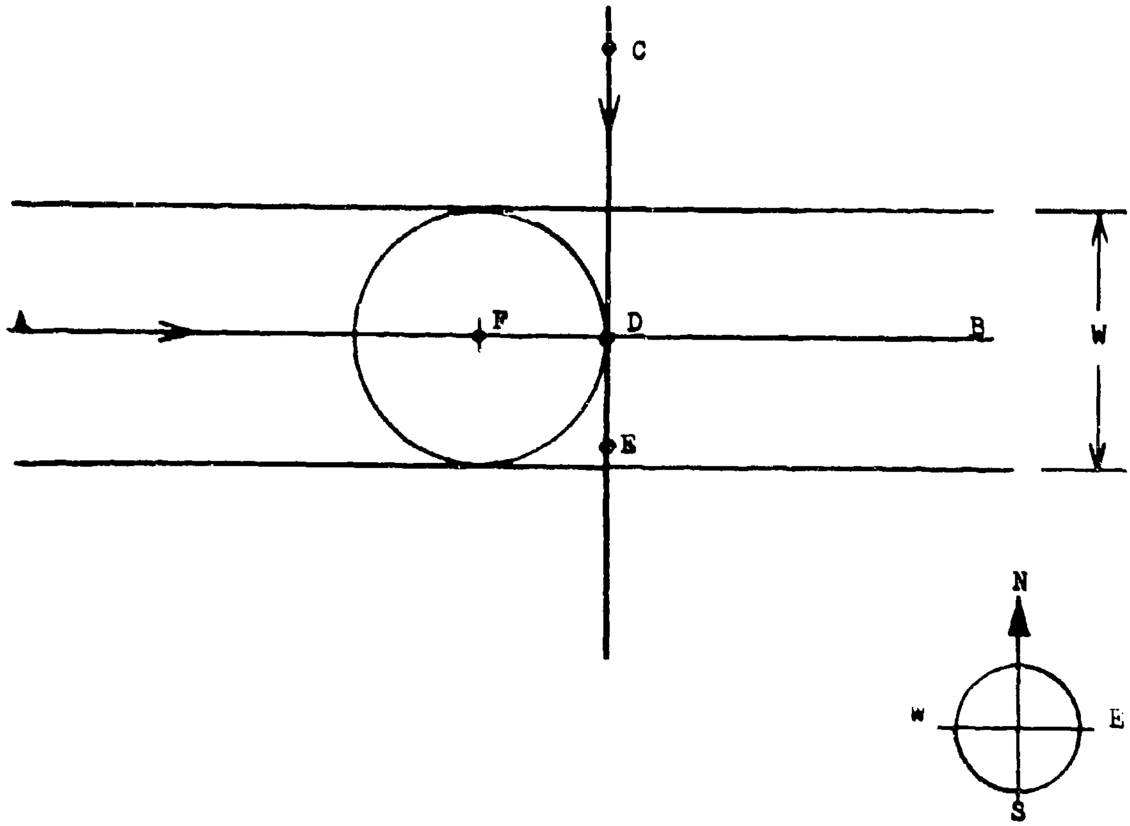


Fig. 4 ATTACK POSITION DIAGRAM

developing a new weapon to increase the patroller's capability. The new weapon is to be a sub-to-sub missile. Each of these weapons imposes a different attack-positioning limitation on the patroller. Of course, the limitations suggested are purely hypothetical. They are intended to illustrate the general approach to the problem, not to represent the limitations of real weapons. It is assumed that if the patroller is using torpedoes, it cannot obtain an attack position unless the initial detection is made while the target is still north of the patrol axis (i.e., the target must originally have been located on the line segment CD). If the target were originally on DE he would be able to avoid the attack. Using the missile, however, the patroller could attack no matter where the target was detected. Let the line segments CD equal T, and DE equal L. Thus the total line segment CDE equals T+L. Then the probability of making a torpedo attack, given a detection, will simply be the ratio of the line segments:

$$Pa = \frac{T}{T+L}$$

The probability of making a missile attack will be one (i.e., Pa=1). It is necessary therefore to determine the length of the line segments L and T.

37. Consider figure 5 below taken from the relative tracks of figure 2 on page 16.

It can be easily seen that

$$\cos\beta = \frac{W}{T+L}$$

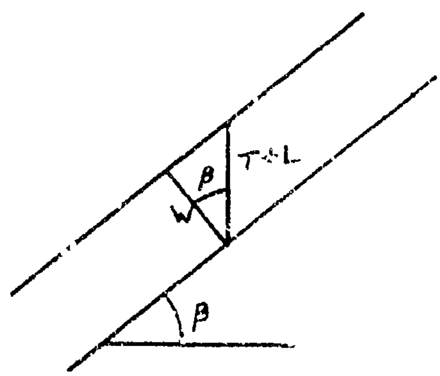


Fig. 5 TARGET TRANSIT-LINE INTERCEPT SEGMENT

$$T+L = \frac{W}{\cos\beta}$$

but $$\tan\beta = \frac{u}{v} = \frac{1}{\lambda}$$

therefore

$$\cos\beta = \frac{v}{(u^2+v^2)^{1/2}} = \frac{1}{\left(\frac{1}{\lambda^2}+1\right)^{1/2}}$$

and $$T+L = W\left(\frac{1}{\lambda^2}+1\right)^{1/2}$$

38. With the total line segment T+L determined it now remains to determine either T or L. L seems to be the easiest to obtain. Consider the diagram of figure 6: Consider the intersection of the patrol axis AB and the target transit path CDE (i.e. the point D) as the origin for all measurements. Then as the patroller at point F in figure 4 advances to F' its detection circle crosses the transit path by a distance x as noted on figure 6. Simultaneously it sweeps a distance y along the southern portion of the transit axis. The quantities x and y can be related by the Pythagorean Theorem:

$$\left(\frac{W}{2}\right)^2 = y^2 + \left(\frac{W}{2} - x\right)^2$$

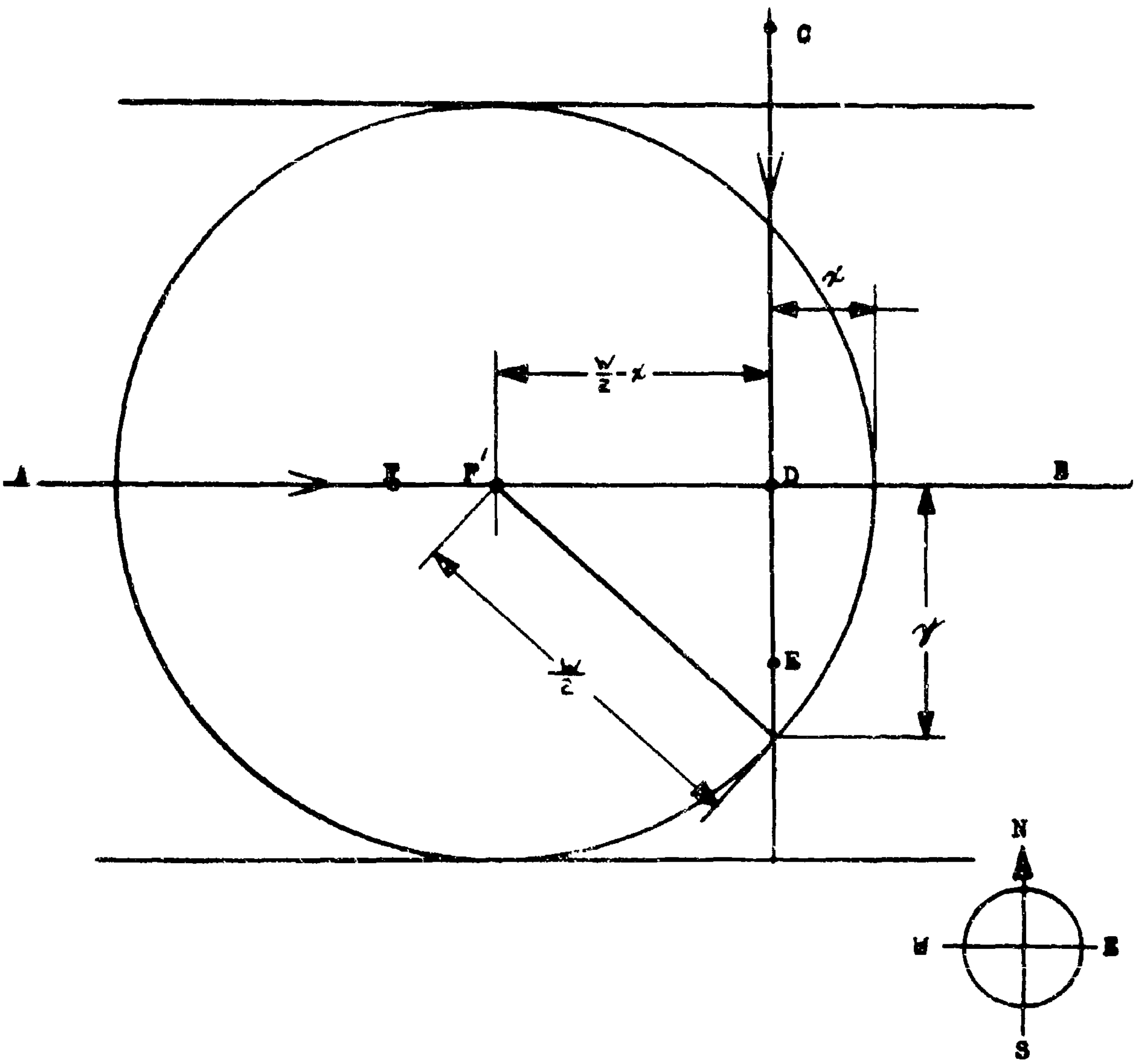


Fig. 6 ATTACK POSITION AFTER AN OBSERVER ADVANCE OF x

$$y^2 = \left(\frac{W}{2}\right)^2 - \left(\frac{W}{2} - x\right)^2$$

$$y^2 = Wx - x^2$$

$$y = \sqrt{Wx - x^2}$$

The negative root has no physical meaning.

While the patroller moves a distance x the target moves a distance $\frac{ux}{v} = \frac{x}{r}$.

If the target is to avoid detection then

$$L + \frac{x}{r} \geq y = \sqrt{Wx - x^2}$$

must hold for all values of x. Suppose

$$L + \frac{x}{r} = \sqrt{Wx - x^2}$$

then

$$\frac{x^2}{r^2} + \frac{2Lx}{r} + L^2 = Wx - x^2$$

$$\left(\frac{1}{r^2} + 1\right)x^2 + \left(\frac{2L}{r} - W\right)x + L^2 = 0$$

Depending on the value of L this equation could have (1) two real but different roots, (2) two real identical roots, or (3) no real roots. If case (1) occurred it would mean that the target had intersected the circular area of detection twice (i.e., $L + \frac{x}{r} < y$ at some point on x) once entering it and the other

leaving it. Case (2) would imply that the target had been tangent with the detection area at one point (i.e., $L+\frac{x}{h}=y$ at one point and $L+\frac{x}{h}>y$ at all other points) but never intersected it. Case (3) would mean that the target avoided the detection area entirely (i.e. $L+\frac{x}{h}>y$ for all values of x). Obviously case (2) is the one of interest; it yields the northernmost position of a target that can escape to the south ahead of the observer submarine. If Case (2) is to occur then by the quadratic formula:

$$ax^2+bx+c=0 \qquad b^2=4ac$$

$$\left(\frac{2L}{h}-W\right)^2=4L^2\left(\frac{1}{h^2}+1\right)$$

$$\frac{4L^2}{h^2}-\frac{4WL}{h}+W^2=\frac{4L^2}{h^2}+4L^2$$

$$4L^2+\frac{4W}{h}\cdot L-W^2=0$$

$$L=\frac{W}{2}\left(\sqrt{\frac{1}{h^2}+1}-\frac{1}{h}\right)$$

Again the negative root has no physical significance since it would make L negative.

39. We can now find T since (T+L) -L = T

$$T=\frac{W}{2}\sqrt{\frac{1}{h^2}+1}+\frac{W}{2h}$$

and the possibility of making a torpedo attack, given a detection is

$$P_a = \frac{T}{T+L}$$

$$P_a = \frac{\frac{W}{2}\sqrt{\frac{1}{r^2}+1} + \frac{W}{2r}}{W\sqrt{\frac{1}{r^2}+1}}$$

$$P_a = \frac{1}{2} + \frac{1}{2r\sqrt{\frac{1}{r^2}+1}} = \frac{1}{2} + \frac{1}{2(1+r^2)^{1/2}}$$

Notice that Pa is independent of W. This is to be expected since the line CDE is proportional to W. As a simple example let us assume that the patrolling and target submarines are moving at the same speed; then r=1 and

$$P_a = \frac{1}{2} + \frac{1}{2(2)^{1/2}} = 0.854$$

40. Of particular interest are the limiting cases when r approaches infinity and zero. Notice that when

$$r \longrightarrow \infty \qquad P_a \longrightarrow \frac{1}{2}$$

since $$P_a \longrightarrow \frac{1}{2} + \frac{1}{\infty} = \frac{1}{2}$$

This corresponds to the case where the target submarine is stationery within the patrol area. Then as the patroller sweeps by, the probability that the target is north of the axis is equal to the probability that it is to be south. and both are equal to one-half. When

$$r \rightarrow 0 \qquad P_a \rightarrow 1$$

since
$$P_a \rightarrow \frac{1}{2} + \frac{1}{2(1+0)^{1/2}} = \frac{1}{2} + \frac{1}{2} = 1$$

This corresponds to the case where the patrolling submarine is stationary within range of the target axis. Of course the target would always first be detected north of the patrol axis. The table below shows how Pa varies as a function of r

r	Pa
0.0	1.000
0.3	0.979
0.5	0.947
1.0	0.854
2.0	0.724
3.3	0.644
∞	0.500

41. The real physical significance of Pa is that it is the probability that the patroller will first detect the target while it is still to the north of the patrol axis. In this hypothetical example Pa has also been taken to be the probability of being able to make a torpedo attack. This latter is a purely arbitrary assumption for illustrative purposes and does not reflect the limitations of real torpedo systems. Real torpedoes do present the attacker with the necessity of maneuvering to a position compatible with their offensive capabilities, however.

42. Let Pc be the probability that the weapon will be capable of destroying the target, given that it functions reliably. For each of the three weapon systems being considered, an arbitrary expression will be assumed for Pc. The purpose here will not be to describe the capabilities of real weapons, but only to indicate the type of tradeoffs with which an analyst may be confronted. Torpedo type I is assumed to be big, powerful, fast, and noisy. For this weapon Pc is assumed to be:

$$P_c(I) = \cos \frac{\pi a}{2T} \qquad 0 \leq a \leq T$$

Recall that T is the vulnerable portion of the target transit path which lies to the north of the patrol axis. Thus the farther north the target lies on line segment T the more effective the torpedo is. The tacit assumption is being made here that the patrollers speed is nearly equal to or greater than the targets speed to assure the reasonableness of the expression for P_c (I). Since the analyst will eventually want to compute the average cost per kill and since the target is assumed to lie at any point on T with equal probability, the average value of P_c (I) will be of interest.

$$\overline{P_c(I)} = \frac{1}{T}\int_0^T \cos\left(\frac{\pi a}{2T}\right) da$$

$$= \frac{1}{T}\left[\frac{2T}{\pi} \sin\left(\frac{\pi a}{2T}\right)\right]_0^T$$

$$= \frac{2}{\pi} = 0.637$$

43. Torpedo type II is assumed to be small, less powerful, and slow, but it is extremely quiet. For this weapon assume

$$P_c(II) = 1 - \sin\left(\frac{\pi a}{2T}\right) \qquad 0 \leq a \leq T$$

The average value of P_c (II) can be found as follows:

$$\bar{P}_c(II) = \frac{1}{T}\int_0^T \left(1 - \sin\left(\frac{\pi a}{2T}\right)\right) da$$

$$= \frac{1}{T}\left[a + \frac{2T}{\pi}\cos\left(\frac{\pi a}{2T}\right)\right]_0^T$$

$$= 1.000 - \frac{2}{\pi} = 0.363$$

44. The proposed missile is expected to be capable of attack not only over the length T but over L also. For both line segments assume this weapon has

$$Pc\ (III) = 1.$$

45. It is not within the scope of this report to discuss the methodology of the weapon reliability Pr. This methodology has been exhaustively discussed in such works as references (d-g). For the two torpedoes of this analysis Pr will be assumed to be known as:

$$Pr = 0.9$$

For the yet undeveloped missile, Pr is a parameter which will depend upon the price the decision-maker is willing to pay for

the weapon. A range of Pr from 0.5 to 0.9 will be considered in relation to cost.

46. The average costs for making a successful attack will now be considered for each weapon and for mixes of the weapons. Of course the costs assigned will be hypothetical and for illustrative purposes only. In this inflationary world any projected costs would soon seem archaic no matter how large the numbers assumed, but the basic mathematical techniques will remain unchanged.

Let:

(a) Kp - be the average cost of operating a submarine on a thirty-day barrier patrol. This figure includes all maintenance, training, depreciation, and other costs.

(b) Kt - be the value of a torpedo-equipped submarine.

(c) Ks - be the value of a submarine equipped with both torpedoes and a missile system.

(d) Kb - be the cost of torpedo type one. This price includes all development, training, maintenance, and exercise costs on a pro rata basis.

(e) Kc - be the similar cost for torpedo type two.

(f) Km - be the similar cost estimate for the proposed missile.

In the case of the missile it will be assumed that the decision-maker can choose between three possible designs with reliability cost tradeoffs as follows:

Pr	Km
0.5	$ 1 000 000
0.7	1 500 000
0.9	3 000 000

A major concern of the decision maker will be to decide whether or not a missile system should be developed and if so which alternative. The other costs will be assumed as follows:

$$\begin{aligned} Kp &= \$\ 1\ 000\ 000 \\ Kt &= \$\ 25\ 000\ 000 \\ Ks &= \$\ 30\ 000\ 000 \\ Kb &= \$\ 200\ 000 \\ Kc &= \$\ 100\ 000 \end{aligned}$$

47. As noted in paragraph 23 the cost-effectiveness (C_k) can be computed as

$$Ck = \frac{Ca}{Pk} + C_r$$

Ck is the average cost-per-kill which is proportional to the cost-effectiveness. Strictly speaking perhaps, cost-effectiveness should be defined as $\frac{1}{c_k}$ but this subtlety will be ignored in this report. The analysts object is to minimize Ck which is synonymous with maximizing cost-effectiveness. Assume that the speed of a transiting submarine has been estimated to be 16 knots and a patrol submarine speed of 16 knots has been selected to counter the threat. Then:

$v = 16$, $u = 16$, $r = 1$, and $Pd = 0.696$

for the torpedoes, $Pa = 0.854$

for the missile, $Pa = 1.000$

It is assumed that if the patroller either doesn't detect the target or detects it but cannot make an attack, the target will not detect the patrolling submarine. If the patroller makes an attack and fails to destroy the target, there will be no opportunity for a second attack, but the target will be able

to make one counterattack. Let Pe be the probability that the counterattack will be successful. Pe will depend on the weapon used in the first attack. Torpedo type (I) is noisy and immediately betrays the position of the friendly submarine. Type (II) is extremely and allows the target little opportunity to determine the patroller's location. The missile will be less certain. It may betray the patroller's position, but not necessarily. Therefore the following is assumed for:

torpedo type (I)	Pe (I) = 0.1
torpedo type (II)	Pe (II) = 0.01
missile	Pe (III) = 0.05

It is also assumed that the average number of enemy transits per thirty-day patrol is one.

48. As noted in paragraph 23

$$Pk = Pd \,.\, Pa \,.\, Pc \,.\, Pr$$

Ca can be found simply as

$$Ca = K_p + K_1$$

where Kl is cost of the patricular weapon used (i.e., K_b, K_c, or K_m), and Cr can be found as follows:

for torpedoes $$C_r = (P_d \cdot P_a)(1 - P_c \cdot P_r)(P_e)\,K_t$$

for the missile $$C_r = (P_d \cdot P_a)(1 - P_c \cdot P_r)(P_e)\,K_s$$

where (Pd . Pa) is the probability that the patroller commits itself to an attack; (1 - Pc . Pr) is the probability that the attack fails; Pc is the probability that the enemy succeeds in destroying the patroller in the counterattack; and (K_t or Ks)

is the cost of losing the patrol submarine.

49. The cost-effectiveness C_k can now be computed for each weapon and for mixes of the weapons. First for torpedo type (I)

$$\overline{C_k(I)} = \frac{K_p + K_t}{P_d \cdot P_a \cdot \overline{P_c} \cdot P_k} + (P_d \cdot P_a)(1 - P_c \cdot P_k)(P_e)(K_t)$$

where $\overline{P_c(I)} = 0.637$

Hence: $\overline{C_k(I)} = \$\,4\,230\,000$

The average cost per kill using torpedo type (I) alone.

For torpedo type (II)

$$\overline{C_k(II)} = \frac{K_p + K_t}{P_d \cdot P_a \cdot \overline{P_c} \cdot P_k} + (P_d \cdot P_a)(1 - P_c \cdot P_k)(P_e)(K_t)$$

$$\overline{C_k(II)} = \$\,7\,570\,000$$

For the three missile alternatives:

$$\overline{C_k(III)} = \frac{K_p + K_m}{P_d \cdot P_a \cdot \overline{P_c} \cdot P_k} + (P_d \cdot P_a)(1 - P_c \cdot P_k)(P_e)(K_s)$$

FOR $P_k = 0.5$ $\quad \overline{C_k(III)} = \$\,6\,270\,000$

FOR $P_k = 0.7$ $\quad \overline{C_k(III)} = \$\,5\,440\,000$

for Pr = 0.9

$\overline{C_k\ (III)} = \$6\,490\,000$

Clearly if a missile is to be developed it should be the alternative with Pr = 0.7. Unless, of course, there are non-quantifiable

factors which the decision maker must consider along with C_k. Notice that for the torpedoes Ck is an average value. For any particular situation the value of Ck depends on the position of the target when first detected on the line T+L. But for the missile this is not the case. Ck is the same no matter where the target is first detected.

51. As was noted in paragraph 49, torpedo type (I) has a much lower cost-per-kill than type (II) on the average. But this is not true for all values of a. For example, when a=o (i.e., the target is located at point C of figure 4):

C_k (I) = \$ 2 390 000

and

C_k (II) = 2 070 000

In this situation, torpedo type (II) offers a small advantage over type (I). There exists a point such that Ck (I) = Ck (II). For all values of (a) less than this point torpedo type (II) should be used. Beyond it torpedo type (I) should be used. Thus a mix of torpedoes should yield a slightly lower $\bar{C_k}$ than $\overline{C_k}$ (I). The crossover point can be found most easily by simple brute force cut-and-try methods with or without the use of a computer. The crossover in this case occurs when

a = 0.09T

Therefore the appropriate $\overline{Pc}$'s must be recalculated.

$$\overline{P_c(I)} = \frac{1}{0.91T}\int_{0.09T}^{T} \cos\left(\frac{\pi a}{2T}\right) da$$

$$\overline{P_k(I)} = 0.601$$

and the new $\overline{C_k(I)} = \$\,4\,370\,000$

similarly

$$\overline{P_k(II)} = \frac{1}{0.09T}\int_0^{0.09T}\left[1 - \sin\left(\frac{\pi a}{2T}\right)\right] da$$

$$\overline{P_k(II)} = 0.920$$

and

$$\overline{C_k(II)} = \$\,2\,160\,000$$

52. The cost-effectiveness $\overline{Ck}$ for the torpedo mix is then:

$$\overline{C_k} = 0.91\,C_k(I) + 0.09\,C_k(II)$$

$$\overline{C_k} = \$\,4\,150\,000$$

and a small advantage over the use of torpedo type (I) is realized when this result is compared with that in paragraph 49. At first glance the improvement may seem so small that the existence of type (II) seems unjustified. But notice that the use of type II greatly reduces the risk to the friendly submarine, and this means that fewer lives are lost. Thus torpedo type (II) has a major benefit which is not quantifiable and so it may frequently be used in situations where its cost-effectiveness is less favorable than that of torpedo type (I).

53. If the missile with Pr = 0.7 were developed and added to the mix would it create a more favorable $\overline{Ck}$? Certainly at values of (a) greater than (T) it would contribute to the patrollers

effectiveness since the patroller has no capability in the range

$$T \leq a \leq T+L$$

if only torpedoes are used. Again a crossover point exists between torpedo type (I) and the missile. Once more, by cut-and-try methods, the following crossover is found:

$$s = 0.68T.$$

54. Notice that since

$$Pa = \frac{T}{T+L} = 0.854$$

$$L = 0.171T$$

and the total length of the detection line segment is

$$T+L = 1.171T$$

Therefore, $\overline{Ck}$ for a torpedo plus missile mix can be expressed as

$$\overline{C_K} = \left(\frac{0.68-0.09}{1.171}\right)\overline{C_K(I)} + \left(\frac{0.09}{1.171}\right)\overline{C_K(II)} + \left(\frac{0.491}{1.171}\right)\overline{C_K(III)}$$

$$\overline{C_K} = 0.504\,\overline{C_K(I)} + 0.077\,\overline{C_K(II)} + 0.419\,\overline{C_K(III)}$$

where

$$\overline{P_K(I)} = \frac{1}{0.59T}\int_{0.09T}^{0.68T} \cos\left(\frac{\pi a}{2T}\right) da$$

$$\overline{P_K(I)} = 0.794$$

$$\overline{C_K(I)} = \$\ 3\ 250\ 000$$

$$\overline{C_K(II)} = \$\ 2\ 160\ 000$$

$$\overline{C_K(III)} = \$\ 5\ 440\ 000$$

for the total mix

$$\overline{Ck} = \$ \; 4\,080\,000$$

which is a disappointingly small improvement over the value computed in paragraph 52. Other factors being equal, a decision maker would be reluctant to develop a weapon that offered such a small advantage.

55. The major drawback to the missile system is that its cost is so unusually high when compared with the worth of the patrol submarine. In real life, the submarine worth is probably mugh higher and this would favor the development of the missile. But this example in Part II was meant only to illustrate the general methods that an analyst must use in solving an (OA) problem. The values used were hypothetical, but the illustration does serve to indicate the complexity that can arise from a relatively simple tactical situation. The need for a systematic application of the methods outlined in Part I should now be obvious.

```
54692A B6NØL
Ø

 TELETYPE NUMBER--- 12


USER NUMBER--M10730

*** ØN AT  14:09.

SYSTEM--BASIC
NEW ØR ØLD--ØLD
ØLD PRØBLEM NAME--100076
WAIT.

READY.

LIST

 100076     14:09    CEIR    12/13/68

10 FØR I=1 TØ 8
20 READ V(I)
30 NEXT I
40 DATA 6,8,10,12,14,16,18,20
50 FØR J=1 TØ 8
60 READ U(J)
70 NEXT J
80 DATA 6,8,10,12,14,16,18,20
90 FØR I=1 TØ 8
100 FØR J=1 TØ 8
110 LET D=50
120 LET A=-.588
130 LET B=2.176
140 LET C=(((D/(V(I)*A+U(J)*B)-1)-(SQR((V(I)/U(J))↑2+1)-1)/2)↑2)
150 LET G=((D/(V(I)*A+U(J)*B))-1)*(D/(V(I)*A+U(J)*B))
155 LET W=A*V(I)+B*U(J)
160 LET P=1-C/G
170 PRINT "P="P, "V="V(I), "U="U(J), "W="W
180 NEXT J
190 NEXT I
200 END


RUN
WAIT.
```

Enclosure (1)

100076 14:10 CELK 12/13/68

P= .267568	V= 6	U= 6	W= 9.528
P= .345333	V= 6	U= 8	W= 13.88
P= .423795	V= 6	U= 10	W= 18.232
P= .503698	V= 6	U= 12	W= 22.584
P= .584893	V= 6	U= 14	W= 26.936
P= .667102	V= 6	U= 16	W= 31.288
P= .750063	V= 6	U= 18	W= 35.64
P= .833508	V= 6	U= 20	W= 39.992
P= .274678	V= 8	U= 6	W= 8.352
P= .355611	V= 8	U= 8	W= 12.704
P= .43337	V= 8	U= 10	W= 17.056
P= .511319	V= 8	U= 12	W= 21.408
P= .59023	V= 8	U= 14	W= 25.76
P= .670149	V= 8	U= 16	W= 30.112
P= .750891	V= 8	U= 18	W= 34.464
P= .832125	V= 8	U= 20	W= 38.816
P= .273599	V= 10	U= 6	W= 7.176
P= .362842	V= 10	U= 8	W= 11.528
P= .442814	V= 10	U= 10	W= 15.88
P= .520465	V= 10	U= 12	W= 20.232
P= .597998	V= 10	U= 14	W= 24.584
P= .676068	V= 10	U= 16	W= 28.936
P= .754731	V= 10	U= 18	W= 33.288
P= .833671	V= 10	U= 20	W= 37.64
P= .262078	V= 12	U= 6	W= 6
P= .364537	V= 12	U= 8	W= 10.352
P= .449692	V= 12	U= 10	W= 14.704
P= .528918	V= 12	U= 12	W= 19.056
P= .606245	V= 12	U= 14	W= 23.408
P= .683172	V= 12	U= 16	W= 27.76
P= .760133	V= 12	U= 18	W= 32.112
P= .836914	V= 12	U= 20	W= 36.464
P= .238826	V= 14	U= 6	W= 4.824
P= .35926	V= 14	U= 8	W= 9.176
P= .452466	V= 14	U= 10	W= 13.528
P= .535156	V= 14	U= 12	W= 17.88
P= .613546	V= 14	U= 14	W= 22.232
P= .690169	V= 14	U= 16	W= 26.584
P= .765957	V= 14	U= 18	W= 30.936
P= .840884	V= 14	U= 20	W= 35.288

```
P= .202888      V= 16       U= 6        W= 3.648
P= .34613       V= 16       U= 8        W= 8
P= .450179      V= 16       U= 10       W= 12.352
P= .538178      V= 16       U= 12       W= 16.704
P= .613912      V= 16       U= 14       W= 21.056
P= .696127      V= 16       U= 16       W= 25.408
P= .771362      V= 16       U= 18       W= 29.76
P= .844855      V= 16       U= 20       W= 34.112
P= .153337      V= 18       U= 6        W= 2.472
P= .324513      V= 18       U= 8        W= 6.824
P= .442212      V= 18       U= 10       W= 11.176
P= .537332      V= 18       U= 12       W= 15.528
P= .621678      V= 18       U= 14       W= 19.88
P= .7004        V= 18       U= 16       W= 24.232
P= .775749      V= 18       U= 18       W= 28.584
P= .848305      V= 18       U= 20       W= 32.936
P= 8.91436 E-2              V= 20       U= 6        W= 1.296
P= .293852      V= 20       U= 8        W= 5.648
P= .428115      V= 20       U= 10       W= 10
P= .532179      V= 20       U= 12       W= 14.352
P= .621397      V= 20       U= 14       W= 18.704
P= .702547      V= 20       U= 16       W= 23.056
P= .778704      V= 20       U= 18       W= 27.408
P= .850867      V= 20       U= 20       W= 31.76

TIME:    5 SECS.


SAVE
WAIT.

READY.

BYE

*** ØFF AT 14:18.
```

Pd Matrix for a 50 Nautical Mile Barrier

u (target speed)

v (OBSERVER SPEED)	6	8	10	12	14	16	18	20
6	0.268	0.345	0.424	0.504	0.585	0.667	0.750	0.834
8	0.275	0.356	0.433	0.511	0.590	0.670	0.751	0.832
10	0.274	0.363	0.443	0.520	0.598	0.676	0.755	0.834
12	0.262	0.365	0.450	0.529	0.606	0.683	0.760	0.836
14	0.239	0.359	0.458	0.535	0.614	0.690	0.766	0.841
16	0.203	0.346	0.450	0.538	0.619	0.696	0.771	0.845
18	0.153	0.325	0.441	0.537	0.622	0.700	0.776	0.848
20	0.089	0.294	0.428	0.532	0.621	0.703	0.779	0.851

Enclosure (2)

Pd Matrix for a 100 Nautical Mile Barrier

u (target speed)

v (own sub speed)	6	8	10	12	14	16	18	20
6	0.134	0.173	0.212	0.252	0.293	0.334	0.376	0.417
8	0.138	0.179	0.218	0.257	0.296	0.336	0.377	0.418
10	0.138	0.183	0.223	0.262	0.301	0.340	0.380	0.420
12	0.133	0.185	0.228	0.268	0.307	0.345	0.384	0.423
14	0.121	0.183	0.230	0.272	0.312	0.351	0.389	0.428
16	0.103	0.176	0.230	0.275	0.316	0.356	0.395	0.433
18	0.077	0.165	0.226	0.275	0.319	0.360	0.399	0.438
20	0.045	0.150	0.219	0.274	0.320	0.363	0.403	0.443

Pd Matrix for a 200 Nautical Mile Barrier

u (target speed)

v (own sub speed)	6	8	10	12	14	16	18	20
6	0.067	0.087	0.106	0.126	0.146	0.167	0.188	0.209
8	0.069	0.090	0.109	0.129	0.148	0.168	0.188	0.209
10	0.069	0.092	0.112	0.131	0.151	0.170	0.190	0.210
12	0.067	0.093	0.114	0.134	0.154	0.173	0.193	0.212
14	0.061	0.092	0.116	0.137	0.157	0.176	0.195	0.215
16	0.052	0.089	0.116	0.13	0.159	0.178	0.198	0.218
18	0.039	0.083	0.114	0.139	0.161	0.181	0.201	0.221
20	0.022	0.075	0.111	0.136	0.162	0.183	0.204	0.223

```
LIST
WAIT.


 GE0076     15:24    CEIR    01/22/69

10   LET D=50
20   LET A=-0.588
30   LET B=2.176
40   LET U=4
50   LET U=U+2
60   LET V=5
70   LET V=V+1
75   LET E=A*V+B*U
80   LET F=D/E
90   LET G=SQR((V/U)↑2+1)
100 LET D1=(F-1)*F*2*(F-1-(G-1)/2)
101 LET E1=-(F*A/E)-(V/U↑2/(2*G))
102 LET H1=D1*E1
110 LET H2=((F-1)-((G-1)/2))↑2*(((F-1)*(-F*A/E))+(F*(-F*A/E)))
120 LET H3=((F-1)*F)↑2
130 LET P1=(H1-H2)/H3
140 PRINT "U="U,"V="V,"P1="P1
150 IF V<20 THEN 70
160 IF U<20 THEN50
170 END


BYE

*** OFF AT 15:25.
```

ENCLOSURE 3

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(g) NAVWEPS 00-65-502, "Handbook Reliability Engineering", 1 Jun 1964 Unclassified

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13. ABSTRACT

A review of the methodology of operations analysis (OA) has been completed. The basic steps required to formulate and solve an OA problem have been listed and discussed in detail. These procedures have been applied to a typical tactical situation — the submarine barrier patrol. An effectiveness model for a submarine using a hypothetical mix of weapons has been generated. Kill probabilities and cost-effectiveness comparisons have been made for a variety of weapon mix possibilities.

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Cost Effectiveness						
ASW						
Barrier Patrol						
Systems Analysis						